

Charging of space	debris and	l their dynar	mical co	nsequences

Abhijit Sen University of Calcutta

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14. ABSTRACT

13. SUPPLEMENTARY NOTES

The charging of space debris due to the ambient plasma environment in the low earth orbit (LEO) and geostationary earth orbit (GEO) regions of the ionosphere has been investigated using both analytic and particle-in-cell (PIC) modeling. The analytic estimates have been obtained using improved Orbit Motion Limited (OML) modeling while the simulation studies have been carried out using the open source simulation code SPIS. In the GEO region account has been taken of charging arising from photoemission effects as well as due to the impact of energetic charged particle beams associated with solar flares. The PIC approach has also been used to study differential charging of debris objects that are composed of patches of conducting and insulated regions. For debris sizes larger than a few microns the orbital perturbations arising from the interaction of the debris charge with the ambient magnetic field and ionospheric electric fields are found to be insignificant in comparison to those arising from gravitational and solar radiation pressure effects and hence may have a negligible influence on orbital calculations. However the debris charge can give rise to nonlinear electrostatic wave excitations in the ambient plasma in the form of precursor solitons that move ahead of the debris. A model calculation delineating the conditions and nature of such excitations has been carried out. A controlled laboratory experiment to test this theoretical concept has also been done and has shown the first ever excitation of precursor solitons in a plasma medium. Detection of such waves in the debris orbital regions using ground-based techniques can provide a novel method of detecting centimeter-sized objects that are otherwise difficult to track using optical methods. The discovery of these precursor excitations also opens up potential new areas of fundamental and applied research in the field of plasmas and space physics.

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Principal Investigator: Prof. Abhijit Sen

Affiliation: Institute for Plasma Research

Mailing Address: Near Indira Bridge, Bhat,

Gandhinagar 382428, India

Telephone: +91-9825051578

Fax: +917923969016

Email: senabhijit@gmail.com

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Abstract

The charging of space debris due to the ambient plasma environment in the low earth orbit (LEO) and geostationary earth orbit (GEO) regions of the ionosphere has been investigated using both analytic and particle-in-cell (PIC) modeling. The analytic estimates have been obtained using improved Orbit Motion Limited (OML) modeling while the simulation studies have been carried out using the open source simulation code SPIS. In the GEO region account has been taken of charging arising from photoemission effects as well as due to the impact of energetic charged particle beams associated with solar flares. The PIC approach has also been used to study differential charging of debris objects that are composed of patches of conducting and insulated regions. For debris sizes larger than a few microns the orbital perturbations arising from the interaction of the debris charge with the ambient magnetic field and ionospheric electric fields are found to be insignificant in comparison to those arising from gravitational and solar radiation pressure effects and hence may have a negligible influence on orbital calculations. However the debris charge can give rise to nonlinear electrostatic wave excitations in the ambient plasma in the form of precursor solitons that move ahead of the debris. A model calculation delineating the conditions and nature of such excitations has been carried out. A controlled laboratory experiment to test this theoretical concept has also been done and has shown the first ever excitation of precursor solitons in a plasma medium. Detection of such waves in the debris orbital regions using ground-based techniques can provide a novel method of detecting centimeter-sized objects that are otherwise difficult to track using optical methods. The discovery of these precursor excitations also opens up potential new areas of fundamental and applied research in the field of plasmas and space physics.

Work done report

The main objectives of this study have been to estimate the amount of charge that debris objects of various sizes and composition can acquire in the LEO and GEO environment of the earth, to assess the potential impact of this charging on their orbital dynamics and to explore possible ways of exploiting their charged state to aid their detection and tracking. In pursuit of these objectives we have

- (i) made analytic and PIC simulation based estimates of debris objects in various size ranges and for different materials and shapes,
- (ii) assessed their impact on orbital dynamics by comparing the electromagnetic forces they experience with the other forces acting on them and
- (iii) proposed a novel method of tracking debris objects by detecting solitonic precursor wave excitations that can occur due to the large amount of charge on them.
- (iv) confirmed the conceptual basis of the precursor excitation in a controlled laboratory experiment.

For details about (i) and (ii) please refer to the technical report entitled:

Abhijit Sen, Sanat Tiwari, Sanjay Mishra, and Predhiman Kaw, "Charging of Space Debris in the LEO and GEO regions" (attached as Annexure I).

For details on (iii) please refer to the publications:

Abhijit Sen, Sanat Tiwari, Sanjay Mishra, and Predhiman Kaw, "*Nonlinear wave excitations by orbiting charged space debris objects*", Advances in Space Research **56**, 429–435 (2015). (attached as Annexure II)

Sanat Kumar Tiwari, and Abhijit Sen, "Wakes and precursor soliton excitations by a moving charged object in a plasma", accepted for publication in Physics of Plasmas. (attached as Annexure III)

For details on (iv) please refer to the publications:

Surabhi Jaiswal, P. Bandyopadhyay, and A. Sen, "Experimental observation of precursor solitons in a flowing complex plasma", submitted to Physical Review Letters. (attached as Annexure IV)

S. Jaiswal , P. Bandyopadhyay , and A. Sen, "Dusty Plasma Experimental (DPEx) device for complex plasma experiments with flow", Review of Scientific Instruments 86, 113503 (2015) (attached as Annexure V)

List of Publications and Significant Collaborations that resulted from the AOARD supported project

- a) Papers published in peer-reviewed journals
 - (i) Abhijit Sen, Sanat Tiwari, Sanjay Mishra, and Predhiman Kaw, "Nonlinear wave excitations by orbiting charged space debris objects", Advances in Space Research **56**, 429–435 (2015).
 - (ii) S. Jaiswal, P. Bandyopadhyay, and A. Sen, "Dusty Plasma Experimental (DPEx) device for complex plasma experiments with flow", Review of Scientific Instruments 86, 113503 (2015)
 - (iii) Sanat Kumar Tiwari, and Abhijit Sen, "Wakes and precursor soliton excitations by a moving charged object in a plasma", accepted for publication in Physics of Plasmas
- b) Papers published in peer-reviewed conference proceedings,

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c) Papers published in non-peer-reviewed journals and conference proceedings,

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- d) Conference presentations without papers,
 - (i) Abhijit Sen, Sanat K Tiwari, Sanjay K Mishra, Predhiman Kaw and A Surjalal Sharma, *Charging of space debris in the LEO and GEO regions*, 40th COSPAR Scientific Assembly, 2-10 August 2014, Moscow, Russia.
 - (ii) A. Sen, S. K. Tiwari, S. K. Mishra, and P. Kaw, *Nonlinear Wave Excitations By Orbiting Charged Space Debris Objects*, 17th International Congress On Plasma Physics 2014, September 15-19, 2014, Lisbon, Portugal.
 - (iii) Abhijit Sen, Sanat Tiwari, Sanjay Mishra and Predhiman Kaw, *Indirect Detection of Charged Space Debris via Nonlinear Wave Excitations*, 1st URSI Atlantic Radio Science Conference (URSI AT-RASC), 18 25 May 2015, Gran Canaria, Spain.
- e) Manuscripts submitted but not yet published,
 - (i) Surabhi Jaiswal, P. Bandyopadhyay, and A. Sen, Experimental observation of precursor solitons in a flowing complex plasma, submitted to Physical Review Letters.

f) List any interactions with industry or with Air Force Research Laboratory scientists or significant collaborations that resulted from this work.

Scientific exchanges via email with the following scientists:

- (i) Dr. Moriba Jah, Air Force Research Laboratory, Kihei, Maui
- (ii) Dr. A.S. Sharma,
 Space and Plasma Physics Group,
 Department of Astronomy,
 University of Maryland,
 College Park, MD, 20742-2421
- (iii) Dr. Carolin Frueh,
 University of New Mexico,
 USA/Air Force Research Laboratory
 (presently at Purdue University)
- (iv) Dr. Ryan M. Weisman,
 Research Aerospace Engineer
 Guidance, Navigation, and Controls Program
 Air Force Research Laboratory (AFRL/RVSV)
 3550 Aberdeen Ave. SE
 Kirtland AFB, NM 87117
- (v) Dr. Dale Ferguson, Air Force Research Laboratory, U.S.A.
- (vi) Dr. Stephen R. Gildea,
 Aerospace Engineer,
 In-Space Propulsion Branch (RQRS)
 Air Force Research Laboratory
 Edwards Air Force Base, CA

Charging of Space Debris in the LEO and GEO regions

Abhijit Sen, Sanat Kumar Tiwari, Sanjay K Mishra, Predhiman Kaw*
Institute for Plasma Research, Bhat, Gandhinagar - 382428, India

Abstract

We investigate the charging of space debris due to the ambient plasma environment in the low earth orbit (LEO) and geostationary earth orbit (GEO) regions of the ionosphere using both analytic and particle-in-cell (PIC) modeling. The analytic estimates are obtained using improved Orbit Motion Limited (OML) modeling while the simulation studies are carried out using the open source simulation code SPIS. In the GEO region account is taken of charging arising from photoemission effects as well as due to the impact of energetic charged particle beams associated with solar flares. Using the PIC approach we also study differential charging of debris objects that are composed of patches of conducting and insulated regions. The dynamical consequences of debris charging on their orbital trajectories and rotational characteristics are found to be insignificant for debris sizes larger than microns due to the predominance of gravitational and other forces acting on them. The charge can however provide a direct means of detecting small sized debris objects by in-situ measurements of the resultant currents.

^{*} senabhijit@gmail.com

I. INTRODUCTION

The near exponential rise of space debris at the satellite orbital altitudes (particularly in the low earth orbit (LEO) region) and the risk they pose for space assets has attracted a great deal of scientific attention for the past several years and is a source of major concern for all nations engaged in space activities [1, 2]. Considerable efforts are therefore being expended into accurate modeling and tracking of these objects [3, 4] and various ideas for the safe removal of these debris are also being explored [5, 6]. A number of databases cataloguing the size, nature and location of these Resident Space Objects (RSO) have been developed that rely on tracking data obtained from ground based and some in-situ measurements from active satellites/space platforms. The dynamical modeling of these objects to calculate their orbital trajectories then allows the prediction of the future evolution of the distribution of these objects in space and assessing the probability of their collisions with active space crafts. An orbiting piece of space debris is subject to a number of perturbative forces including gravity (due to anharmonicity in the earths gravitational field and effects due to other nearby large bodies), radiation pressure, impacts with neutral particles (atmospheric drag), etc. In addition, there can also be an electromagnetic perturbation due to the charged nature of the debris. The debris objects are likely to acquire a large amount of charge due to the ambient plasma environment in the LEO region ($\sim 400 \text{kms}$ to $\sim 1000 \text{kms}$) and the radiation belts in the geosynchronous orbit region. The consequent flow of electron and ion currents on them lead to the accumulation of a large amount of surface charge and the development of a surface potential on these objects [7]. The influence of the plasma environment on the dynamics and charging of the debris is a relatively unexplored area of Space Situational Awareness (SSA) and Space Debris (SD) research and can be potentially important for the accurate prediction of the long-term evolution of debris orbits and their collision probabilities with other space objects.

The phenomenon of plasma induced charging of space objects has been known for a long time and has been well studied in the past. For example, the lunar surface, in an airless environment, experiences charging due to exposure to solar UV radiation and plasma of solar wind and plasma of magnetospheric and ionospheric origin [8, 9]. Likewise, space crafts develop significant surface potential due to charging from the surrounding plasma en-

vironment. The associated technical problems (e.g. arcing) that they pose for the satellite or for the placement of measurement instruments (e.g. electrostatic probes) etc. are well documented and have also been experimentally verified [10]. Plasma charging has also been exploited in novel proposed applications such as the tether braking scheme for satellites [5, 11]. However the influence of charge on the orbital dynamics of an object becomes most pronounced for very small sized objects for which the ratio of charge to mass is not insignificant and the electromagnetic perturbations become comparable to other perturbative forces that influence the particle trajectory. These small sized particles (< 10cm) are also those that cannot easily be detected by optical and other means and hence an accurate modeling of their dynamics and distribution is very important for predicting their long time evolution and collisional behaviour. For micron sized charged dust particles, it has been shown for example, that their orbits degrade due to the electromagnetic perturbative effects in the magnetosphere forcing them either into elliptic orbits or ejecting them into interplanetary space [12].

In this report we present theoretical estimates of the electrical charge and electrostatic surface potentials on debris objects orbiting in the LEO and GEO regions of the earth's ionosphere. The estimates are obtained using analytic calculations based on the Orbit Limited Model (OML) as well as through numerical PIC simulations using the open source SPIS code. A variety of shapes, sizes and material compositions of the debris objects are considered including so called HAMR objects that have a high area to mass ratio as well as objects composed of patches of insulating and conducting material. In the GEO region account is taken of charging arising from photo-emission effects and also due to the impact of energetic charged particle beams associated with solar flares. The dynamical consequences of debris charging are investigated by comparing the influence of the Lorentz force with other forces acting on the debris objects. The report is organized as follows.

II. ANALYTIC MODEL BASED ESTIMATES

A debris object orbiting in the LEO/GEO space environment is subject to constant bombardment of electrons and ions from the ambient plasma medium and gets charged in the process. A dynamic steady state defined by the balance of electron and ion currents flowing on to its surface determines the floating potential of the debris object. Since electrons are more mobile than ions the debris particles normally acquire a high negative charge due to higher accretion of electrons. However electron emission from the object induced by photo-electric, thermionic, electric field and other secondary processes can reduce the negative charging or even lead to a positive charging of the object. One of the simplest and widely used model for determining the floating potential and the amount of charge on an object immersed in a plasma is the orbit limited model that has been extensively employed in dusty plasma studies. We begin our investigation of debris charging by employing this model for some typical simple shapes and sizes of debris objects in the LEO and GEO regions.

A. Charging of spherical objects in the LEO region

The LEO region, roughly defined to lie between the altitudes of ≈ 400 kms to ≈ 1100 kms above the earth's surface is known to be densely populated with debris objects varying in size from a few microns (mainly from rocket exhaust to fragmented paint flakes) to several centimeters (destroyed satellite parts, abandoned tools etc.) and these objects move at an orbiting speed of about 10 km/s. The ambient day time and night time plasma conditions for various altitudes of this region have been widely studied and the parameters have been well documented by Gurevich [13]. We consider a spherical conducting object (to model a debris object) in this environment and study the charging process due to the accretion of electrons and ions from the plasma and also due to electron emission resulting from the incident monochromatic Lyman- $\alpha(\lambda = 121.57nm)$ radiation - the dominant EUV radiation of the solar spectra at this altitude [14]. From the OML model, the equation governing the charging of the spherical object is given by the current balance equation,

$$\frac{dQ}{dt} = I_{ee} + \eta_i I_{ic} + \eta_e I_{ec} \tag{1}$$

where $Q = z_d e$ is the total charge on the object, e is the electronic charge and z_d is the number of electronic charges. The charging currents on the right hand side (RHS) of (1) are the photoemission current (I_{ee}) , the ion current (I_{ic}) and the electron current (I_{ec}) respectively. The factors $\eta_e(\eta_i)$ represent the sticking coefficients of electrons (ions) over the surface. The above equation can also be rewritten in terms of particle fluxes f_{ij} $(I_{ij} = e\pi a^2 f_{ij})$ where ais the radius of the spherical object to get,

$$(dz_d/dt) = \pi a^2 [f_{ee}(z_d) + \eta_i f_{ic}(z_d) - \eta_e f_{ec}(z_d)]$$
 (2)

The expressions for f_{ee} , f_{ec} and f_{ic} in the case of spherical targets are given by OML theory as,

for
$$z_d < 0$$

$$f_{ec} = n_e \left(\frac{8k_B T_e}{\pi m_e}\right)^{1/2} \exp\left(z_d \alpha_e\right)$$
 (3)

$$f_{ic} = n_i \left(\frac{8k_B T_i}{\pi m_i}\right)^{1/2} (1 - z_d \alpha_i) \tag{4}$$

for
$$z_d \ge 0$$

$$f_{ec} = n_e \left(\frac{8k_B T_e}{\pi m_e}\right)^{1/2} (1 + z_d \alpha_e)$$
 (5)

and
$$f_{ic} = n_i \left(\frac{8k_B T_i}{\pi m_i}\right)^{1/2} \exp\left(-z_d \alpha_i\right) \tag{6}$$

where n_j, T_j and m_j represent the number density, temperature and mass of the jth kind of charged particle respectively (here j = e, i stands for electrons and ions) and k_B is the Boltzmann constant. The first two terms on the RHS of (2) describe the increase in the debris positive charge due to electron emission and ion accretion respectively while the last term corresponds to loss of positive charge due to electron accretion over its surface. In the simplest case when electron emission is absent (strictly applicable to the night time environment) the steady state equation for the floating charge can be written as

$$\eta_i f_{ic}(z_d) - \eta_e f_{ec}(z_d) = 0, \tag{7}$$

Using the expressions defined in (3) to (6) in (7) along with the charge neutrality condition of the plasma environment $(n_e \approx n_i)$, the steady state floating potential on a debris object can be determined from the relation,

$$(1 - z_d \alpha_e \tau) = (\tau/\mu)^{1/2} \exp(z_d \alpha_e) \Rightarrow (1 + v_s \tau) = (\tau/\mu)^{1/2} \exp(-v_{se})$$
 (8)

where $v_{se} = (-z_d\alpha_e)$ is the dimensionless electrostatic surface potential, $\alpha_e = (e^2/ak_BT_e)$, $\tau = T_e/T_i$ and $\mu = m_e/m_i$. It may be noted that (8) yields a unique solution for v_{se} that is independent of the size of the object and only depends on the plasma parameters like the electron (ion) temperatures and masses. Further, for a spherical object the surface potential and the charge are related by the simple expression that holds for a spherical capacitor,

$$z_d e = C v_s \tag{9}$$

where $C \approx 4\pi\epsilon_0 a$. Thus using (8) and (9) we can determine both the surface potential and the charge of the spherical object.

We now obtain a quantitative estimate of the surface potential acquired by a small debris particle in the micron range in a plasma environment typical of the mid-altitude (~ 700 km) domain of the LEO region. The plasma in this region is populated by Oxygen (O) atoms with a density of $\approx 10^6/cc$, while the electrons and ions have densities of $\approx 10^4/cc$. Furthermore the temperature of the Oxygen component $T_O \approx 1400K$ while $T_e \approx 2900K$ and $T_i \approx 1800K$ in this region. The values of the sticking coefficients (η_e, η_i) are close to unity and we take them to be unity for the computations. Using this data set and Eq. (8) we estimate the surface potential to be $v_s \approx -3.5 \Rightarrow (z_d e^2/a) \approx -0.9eV$ and using the charge to potential relation (9) the charge comes out to be $Q = z_d e \approx -605e$ (for $a = 1\mu m$). Since from (9) the steady state charge increases linearly with the particle radius so a 1cm particle would acquire a charge equal to $\approx -6 \times 10^6 e$.

As mentioned earlier, the OML theory is one of the simplest analytic models developed for estimating the charge acquired by a macroparticle in a plasma environment. So it is important to discuss the limits of its applicability particularly in the present context. One of the major simplifying assumptions of the OML theory is the neglect of plasma collisional effects which is valid if the mean free path (l_{mfp}) is much larger than the Debye screening (λ) length and other typical system lengths of interest such as the object size a [15]. In a collisional plasma i.e. when the mean free path of the ions becomes comparable to the system scale lengths, ion-neutral collisions may lead to the trapping of ions in the sheath region around the object [15–17]. Such a bunching of trapped ions can lead to an increase of the ion current to the object [17] and thereby influence the charging. However, in the present case, for LEO altitudes, this effect is negligible since the ion mean free paths $(\sim kms)$ in this region are a lot larger than the Debye lengths and the size of the debris objects. Hence the OML approach can work well in this region and can provide a realistic estimate of the charge on the debris objects. As we will later see, the results obtained from OML also agree reasonably well with PIC simulation results which do not depend on these approximations.

B. Influence of photoemission

We now consider the influence of photoemission on the charging process in the LEO region. As stated earlier, the Lyman- α ($\lambda \approx 121.57nm$) radiation is the dominant EUV

radiation at LEO altitudes which can potentially reduce the negative charge from the object through the process of photoemission. The photoemission current can be written as

$$f_{ph}(z_d) = n_p = \chi \Lambda \qquad (for \ z_d < 0)$$

= $n_p \left[\psi[\xi, (z_d + 1)\alpha_d)] / \Phi(\xi) \right] = \chi \Lambda \left[\psi[\xi, (z_d + 1)\alpha_d] / \Phi(\xi) \right] \qquad (for \ z_d \ge 0)$

where Λ is the incident photon flux, $n_p(=\chi\Lambda)$ is the number of the photoelectrons emitted per unit area from the uncharged surface with χ representing the photoelectric yield of the surface material, T_d is the surface temperature of the debris object,

$$\psi[\xi, (z_d + 1)\alpha_d] = (z_d + 1)\alpha_d \ln[1 + \exp(\xi - (z_d + 1)\alpha_d)] + \Phi(\xi - (z_d + 1)\alpha_d),$$
$$\xi = (h\nu - \varphi)/k_b T_d,$$
$$\Phi(\kappa) = \int_0^{\exp \kappa} [\ln(1 + \Omega)/\Omega] d\Omega$$

The steady state charge on the particle when the electron emission is included can be derived from the relation,

$$f_{ph} + f_{ic} - f_{ec} = 0 (10)$$

Following Bauer [14] the incident photon flux Λ of the EUV radiation in the LEO region can be approximately taken to be $\Lambda \approx 3 \times 10^{11}$ photons/cm². Using the plasma parameters stated above for the ~700 km region and using typical material characteristics of metallic debris objects (e.g. Al with a work function $\varphi = 4.06eV$) with $T_d \approx 250$ the steady state potential obtained from (10) turns out to be of the order of $\approx -0.55eV$. It is seen that the surface potential is effectively reduced by a factor of 2 when the photoelectric emission from the surface is taken into account.

A numerical plot of the steady state electrostatic floating potential calculated for a spherical object in the LEO region plasma environment at various altitudes is shown in Fig. 1. It can be seen that in the absence of photoemission the surface potential becomes more negative with increasing altitude both for day and night time conditions. This can be attributed to the decreasing plasma density with altitude. The night time potential is always less negative than the day time potential due to the weaker ionization at night. When photoemission contribution is taken into account, as shown in the day time middle curve, the trend of the potential curve changes at a certain altitude and begins to become less negative at higher altitudes due to the object losing photon induced electrons.

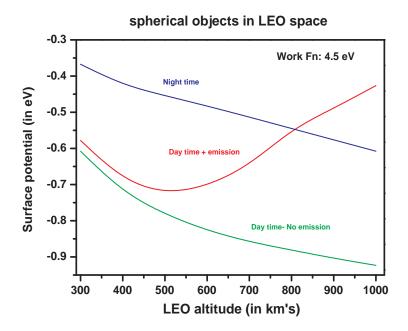


FIG. 1. Debris surface potential as a function of the altitude from the earth in the LEO region.

C. Charging of spherical objects in the GEO region

In the GEO region (at an altitude of 36000 km) the ambient plasma environment is basically established by the solar wind plasma and the characteristic parameters are $n_e \approx 1/cc$, $T_i \approx 10 keV$ and $T_e \approx 3 keV$ [2]. In view of the extremely small magnitudes of the electron/ ion densities the debris charging process is dominated by the photoelectric emission effects due to the incident solar radiation. The photoemission flux from a spherical metallic particle that is exposed to the solar radiation spectrum is given by the following expression that has been derived by Fowler [18],

$$f_{ph}(z_d) = \int_{\varepsilon_{\nu 0}}^{\varepsilon_{\nu m}} \chi(\varepsilon_{\nu}) \left[\psi[\xi, (z_d + 1)\alpha_d] / \Phi(\xi) \right] dn_{inc}, \quad \text{(for } z_d \ge 0)$$
 (11)

$$= \int_{\varepsilon_{\nu 00}}^{\varepsilon_{\nu m}} \chi(\varepsilon_{\nu}) dn_{inc} \quad \text{(for } z_d < 0)$$
 (12)

where $\varepsilon_{\nu_{00}} = \varphi$ is the threshold energy for photoemission, $\varepsilon_{\nu 0} = (\varphi + z_d e^2/a)$ and $\varepsilon_{\nu m}$ are the outer limits of the spectrum. Here dn_{inc} corresponds to the number of photons incident per unit area per unit time due to solar radiation lying in the frequency range ε_{ν} to $(\varepsilon_{\nu} + d\varepsilon_{\nu})$ and can be expressed as

$$dn_{inc} = (r_s/r_d)^2 F(\varepsilon_{\nu}) d\varepsilon_{\nu} = (r_s/r_d)^2 (4\pi\varepsilon_{\nu}^2/c^2) (eh/300)^3 [exp(\varepsilon_{\nu}/k_B T_s) - 1]^{-1} d\varepsilon_{\nu},$$
 (13)

where $r_s (\approx 6.96 \times 10^{10} cm)$ is the radius of the radiating surface of the sun, $r_d (\approx 1.45 \times 10^{13} cm)$

is the mean distance between the sun and the debris ensemble; ε_{ν} is expressed in eV.

In writing the above relation the solar spectrum has been assumed to correspond to that of a black body source radiating at 5800 K. Using the above expression for photoemission rate the steady state charge (potential) on a spherical debris particle in the GEO region can be estimated numerically by solving eq. (10). As a specific example let us consider an Al particle (of work function $\varphi = 4.06eV$) and the solar radiation spectral flux to be in the range 0 < $\varepsilon_{\nu} < 6.5 eV$. The estimated equilibrium floating potential then comes out to be of the order of $\approx 3eV$ (positive, $\chi = 1$) and the corresponding electric charge for a particle with a one micron radius to be $Q=z_de\approx 2016e$. From the scaling relation discussed previously the charge on a 1 cm paricle would be $\approx 2 \times 10^7 e$. The steady state surface potentials acquired by a spherical object at GEO altitudes for different work functions (representing different materials) are plotted in Fig. 2. In these plots we have also tried to account for the spectral dependence of the photoelectric efficiency of a given material by comparing the model dependencies given in the formulations of Spitzer [19] and Draine [20]. Spitzer's formulation considers the photoemission process as a surface phenomenon while Draine's analysis considers bulk processes as well by taking into account complex mechanisms like spectral attenuation and electron escape. The Spitzer formulation is appropriate for photoemission from pure metals while Draine's formulation gives better results for dielectric materials as has been determined from experimental studies [32-33]. Based on such experimental data, the spectral dependence of emission for the two cases can be expressed as,

$$\chi(\varepsilon_{\nu}) = (729\chi_m/16)(\varepsilon_{\nu00}/\varepsilon_{\nu})^4 [1 - (\varepsilon_{\nu00}/\varepsilon_{\nu})]^2 \quad (Spitzer)$$
 (14)

and
$$\chi(\varepsilon_{\nu}) = \chi_m[1 - (\varepsilon_{\nu 00}/\varepsilon_{\nu})], \quad (Draine)$$
 (15)

For our computations we have considered the solar flux lying within the range $0 < \varepsilon_{\nu} < 6.5 eV$. Further the Mie scattering coefficient has been taken to be unity which is justified by the large size of the debris particles compared to the wavelength of the incident radiation [21]. In Fig. (2) we have plotted the surface potential as a function of the work function using both these formulations. As can be seen the surface potential increases almost linearly with decreasing work function except at very low values of the work function where the Spitzer model turns over and the Draine formulation of the photoelectric effect yields estimates for the potential that are larger than the Spitzer model. This deviation can be important for

debris objects that are of mixed composition since it can lead to differential charging of the object.

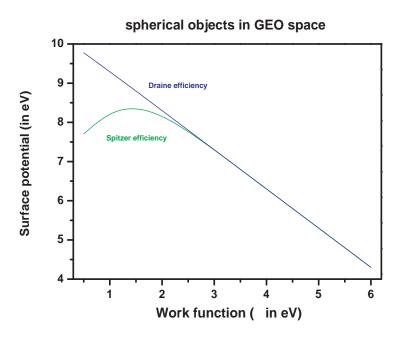


FIG. 2. Object potential as a function of work function in GEO region.

D. Charging of an HAMR object in the GEO region

A class of debris objects, discovered by T. Schildknecht [22] in 2003 and primarily found in the GEO region, are the so called high area-to-mass ratio objects (HAMR) and these have been the subject of many recent studies [23, 24]. The interest in these objects arises from the fact that they are extremely sensitive to orbital perturbations especially of non-conservative forces. For example, solar radiation pressure significantly affects the orbital evolution of these objects even within short time intervals. The radiation pressure force, which depends on the effective area of the debris that is exposed to the Sun, can also exert a torque on the object due to differential pressures arising from partial exposures of the debris surface. Past studies have shown that even small forces and torques have significant effects on the orbit-attitude evolution of HAMR objects. To assess the impact of charging and the concommitant forces arising due to electromagnetic interactions we next estimate the surface potentials and charges acquired by HAMR objects in the GEO region. To model a HAMR debris we consider a plane sheet object and study its charging in a plasma environment of

the GEO region and subject to solar radiation. For the lowest order estimate of the charge and potential we can continue to use the OML model discussed in the previous sections with the proviso that the capacitance expression is changed to that corresponding to a plate capacitor. Since the surface potential remains independent of the size of the object, we can apply our earlier calculated result to an Al plate ($\varphi = 4.05eV$) subjected to a solar radiation flux ($0 < \epsilon_{\nu} < 6.5$) to get an equilibrium surface potential of the order of $\sim 3V$. If the charge density of the plane surface is σ , the surface electric field is given by $E = 2\pi\sigma$. The potential and hence the charge can be written as

$$V_p = -\int_L^0 E dx = -\int_L^0 (2\pi\sigma) dx = 2\pi\sigma L = (2\pi z_d e/L) \Rightarrow z_d = (V_p L/2\pi e)$$
 (16)

If one considers a square plane sheet (as a model approximation of a HAMR) of length 10m, mass $m_d \approx 1$ kg and $\chi = 1$ (max.), the charge comes out to be of the order of $z_d \approx 3 \times 10^9$.

III. PARTICLE IN CELL (PIC) SIMULATION OF CHARGING IN LEO AND GEO REGION

In this section, we report on our 3-D electrostatic PIC simulation studies to estimate the surface potential of model debris objects. PIC simulations offer a direct means of studying the interaction of the debris object with the plasma environment which can serve as a benchmark against which to judge the accuracy of our model calculations discussed in the previous sections. For convenience we once again restrict ourselves to spherical and planar objects to model the debris and study them in the LEO and GEO environment. The simulations have been performed using the Spacecraft Plasma Interaction Software (SPIS) which is available as an open source code [25, 26].

From an operational point of view the code has three major functional sections, namely, (i) Meshing (ii) Numerical Libraries and (iii) post processing or diagnostics. To study the charging of an object, one first forms a mesh of the object using Gmsh tool integrated with the code. One can also design the geometry files externally and then can further import it into the code. Both 2-D and three dimensional grids can be made using Gmsh and then meshed with each cell as a tetrahedron. Further, each meshed object must be assigned minimum three physical groups namely surface group, external boundary group and the

volume group. The physical properties of these groups are then assigned in terms of the required plasma model parameters, material properties, electrical node properties etc. The code is equipped with all NASCAP [27] based material properties and these can also be changed as per requirements. The plasma environment properties (which includes plasma, secondary electron emission, photo-ionization, surface interaction, volume interaction etc.) can be set with the help of a wide range of available parameters. Based on the physics one is interested in, one can model the individual species with a Boltzmann distribution and/or adopt a particle point of view and follow the individual dynamics of each particle. Depending on the choice of a proper representation of the species dynamics the code provides for an appropriate Poisson solver to calculate the fields. In our case, we have evolved the simulations till the object attains a stable surface potential and hence an equilibrium charge density.

Using SPIS, we have carried out charging of spherical and plane sheet type of objects in LEO and GEO regions under various plasma environment conditions. The related results are described in the following subsections.

A. LEO region simulations

The LEO region simulations have been carried out in the altitude range of 400-1000 Km. The plasma environment for these different altitudes have been taken from Gurevich's work [13]. A typical list of parameters used for different the altitudes is tabulated in Table-I.

TABLE I. Typical LEO plasma paramters

Altitude	400 km	600 km	1000 km
Ion species	O^+	O^+	H^+
$T_i(\mathrm{eV})$	0.13	0.18	0.22
$T_e(eV)$	0.21	0.23	0.26
$N_e(m^{-3})$	1.46×10^{12}	3.36×10^{11}	1.4×10^{11}
$N_i(m^{-3})$	1.46×10^{12}	3.36×10^{11}	1.4×10^{11}

Figure 3, shows [L] equilibrium potential of a differentially charged sheet (1cm length

and width with 0.5cm made of each material) made of two different materials Kapton and AlOx and [R] the equilibrium potential obtained by an AlOx sphere of 1cm radius. Both the simulations have been carried out in presence of the sun (i.e. photoelectric effects have been taken into consideration) while the secondary emission is ignored. The simulations have been carried out for 400 Km altitude plasma environment. We see that the spherical object made of Alox has been charged homogeneously due to its conducting surface while the dielectric sheet made of two different materials in fig. 3[L] gets charged differentially. At 400 Km altitude plasma environment, the 1cm spherical Alox debris object gets charged in 10s of microseconds and attains the potential of -0.73V. In the same plasma environment, the plane sheet made of two different materials gets charged -0.84 (Kapton edge) and -0.78 (AlOx edge).

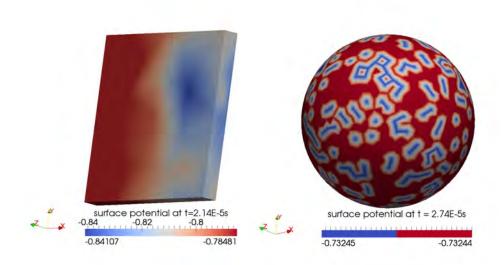


FIG. 3. Equilibrium potential of a [L] differentially charged sheet made of two different materials Kapton and AlOx [R] spherical object made of AlOx. The plasma environment chosen for simulation is LEO region at 400 Km altitude.

B. GEO region simulations

The simulations for the GEO region are carried out for the altitude region of 36000 Kms. In Fig. 4, we have shown the equilibrium surface potential of a 1m spherical object

which is consists of two hemispheres made of different materials viz. KAPT and CERS. These materials are commonly used in manufacturing spacecraft component/objects. The simulations have been carried out for a set of parameters that represent an extreme plasma environment in the GEO region - the so-called NASA worst case parameters provided in the SPIS package with typical plasma parameters $N_e \approx 1.12 \times 10^6/m^3$, H^+ ion species, $N_i \approx 2.36 \times 10^5/m^3$, $T_e \approx 12000 eV$, $T_i \approx 29500 eV$ etc. The electrons have been taken to have a Boltzmannian distribution while the ions are treated as PIC particles. The H^+ ion species are considered to be predominantly present at GEO altitudes.

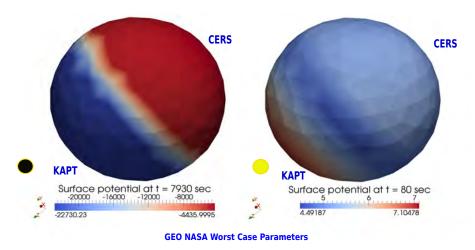


FIG. 4. Equilibrium surface potential of 1m radius shperical object in GEO environment through differential charging [L] in abscence of sun light [R] in presence of sunlight. The sphere is made of two different materials KEPT and CERS.

It is worthwhile at this point to make a comparison of the charging results obtained from the analytic calculations of section ?? with those provided by the PIC simulations discussed in this section. Typical comparisons are shown in Table II displayed below:

TABLE II. Surface potential: Analytical vs Numerical

Altitude	$400 \ km \ (LEO)$	$600 \ km \ (LEO)$	$\boxed{1000~km~(LEO)}$
SPIS-PIC	-0.73 V	-0.76 V	-0.51 V
Analytical	-0.66 V	-0.68 V	-0.45 V

As can be seen the SPIS values are very close to the analytic estimates and hence the analytic values can be used as a good approximation to the expected experimental values

of charge and surface potentials on the debris objects. Using such values we will discuss the dynamical consequences of charging in the next section.

IV. DYNAMICAL CONSEQUENCES OF DEBRIS CHARGING

A charged debris object will be subject to electromagnetic perturbations arising from interactions with the earth's magnetic field as well as with any electric and magnetic fields present due to ionospheric irregularities. A primary question to ask is whether such interactions can in any way have a significant impact on the orbital characteristics of the debris objects. In other words can they accelerate or decelerate the object to change its orbital height. The predominant force on a debris object is the gravitational force which is countered by the centripetal force that keeps the debris in orbit. The high speeds of the debris objects in the range of a few kms/sec provides them with a tremendous kinetic energy which gives the orbits a great deal of dynamical rigidity. So most other perturbations including variations in the earth's magnetic field, radiation pressure due to solar radiation etc. act on very long time scales unless the mass of the debris is very small. To assess the influence of the charging of the debris on its dynamics we can compare the magnitude of the electromagnetic forces acting on it with other forces that it experiences.

We have summarized our findings of such a comparison in Table III where the sizes of the debris, their estimated charges and the forces experienced by them in the LEO and GEO regions are shown. The symbols F_g , F_L , F_{rad} denote gravitational force, Lorentz force and radiation force respectively.

As can be seen the Lorentz force for the charged debris is several orders of magnitude smaller than the gravitational force and also much smaller than the radiation force. So the influence of the Lorentz force on the orbit of most debris objects will be negligible. It is also unlikely to have a significant influence on the rotation characteristics or the orientation of a planar (HAMR) object that has been differentially charged in comparison to that brought about by the radiation force. The basic physics behind this negligible effect of the Lorentz force lies in the smallness of the parameter Q/m - although Q is large the mass is substantially larger compared to an electron or ion mass and hence pulls down this force. For micron or sub-micron size particles however the effect can be important as has been noted before [12]. We can also estimate the acceleration due to the ambient electric field

TABLE III. Force comparison for different debris sizes

	LEO Region			GEO Region			Mean altitude
Debris rad. (μm)	600 Km			36000 Km			
	$F_g(N)$	$z_d(e)$	$F_L(N)$	$F_g(N)$	$z_d(e)$	$F_L(N)$	$F_{rad}(N)$
0.01	1.2×10^{-19}	4.89	3.1×10^{-19}	2.6×10^{-21}	1.4	1.2×10^{-21}	2.1×10^{-21}
0.1	1.2×10^{-16}	48.9	3.1×10^{-18}	2.6×10^{-18}	1.4×10^2	1.2×10^{-20}	2.1×10^{-19}
1	1.2×10^{-13}	4.9×10^2	3.1×10^{-17}	2.6×10^{-15}	1.4×10^3	1.2×10^{-19}	2.1×10^{-17}
10	1.2×10^{-10}	4.9×10^3	3.1×10^{-16}	2.6×10^{-12}	1.4×10^4	1.2×10^{-18}	2.1×10^{-15}
100	1.2×10^{-7}	4.9×10^4	3.1×10^{-15}	2.6×10^{-9}	1.4×10^5	1.2×10^{-17}	2.1×10^{-13}
1000	1.2×10^{-4}	4.9×10^5	3.1×10^{-14}	2.6×10^{-6}	1.4×10^6	1.2×10^{-16}	2.1×10^{-11}
10000	0.118	4.9×10^6	3.1×10^{-13}	0.0026	1.4×10^7	1.2×10^{-15}	2.1×10^{-9}

fluctuations in the plasma environment. The maximum plasma field that one can expect in the GEO region can be roughly estimated by comparing the field energy with the plasma energy density. This would give us

$$E_o \approx (4\pi n_e T)^{1/2} \approx 10^{-5} esu = 0.003 V/cm$$

In the presence of this fluctuating electric field the object can gain an acceleration of $a_d = (z_d e E_o/m_d) \approx 10^{-9} cm/s^2$ which again indicates that for any debris object larger than a micron size the resultant deviation in the orbit would be insignificant.

Although its impact on the orbital dynamics is inconsequential, the charge on a debris object can lead to other phenomena that can have important implications for debris detection and tracking. A novel concept based on tracking precursor nonlinear excitations ahead of the debris trajectory was proposed and discussed in [28]. Here we discuss another concept that may provide a means of in-situ detection of small sized debris in the vicinity of a space-craft. Since the charged moving debris constitutes a current and an attendant local magnetic field the basic idea is to detect and measure this current by measuring the change in magnetic flux as the debris passes through a coil. This adoption of a standard Rogowski coil configuration attached on a boom to the space-craft can provide a non-destructive means

of detecting space debris in close proximity to the craft. To estimate the magnitude of the current consider a 10 cm object in the GEO region, which from our earlier calculations would carry a charge $q \sim 10^9 e$. When the object passes through the coil at a velocity of the order of 8km/sec the pulsed current that the coil senses is $I = q/\Delta t \sim 100nA$ (lower estimate), where Δt refers to the time duration for which the coil effectively feels the influence of the changing magnetic flux. Such a magnitude of current is within sensing limits of a Rogowski coil with a radius of a few meters and that can be mounted with a boom on the space craft. A conceptual schematic is shown in Fig. 5. The concept needs to be tested on a scaled down version in the laboratory and the suitability of various coil materials keeping weight and structural strength need to be examined.

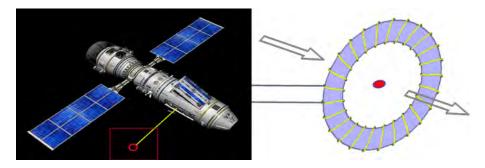


FIG. 5. A schematic showing possible use of Rogowsky coil as non-destructive sensor for small size (mm/cm) debris.

V. SUMMARY AND CONCLUSIONS

The charging of space debris under the influence of the ambient plasma environment and solar radiation has been studied for objects in the LEO and GEO regions. Analytic estimates of the charge and the surface floating potential on debris objects of various sizes and material composition have been obtained using the OML model. In addition, particle-in-cell simulations have been used to obtain a more direct estimate of these quantities and the results are found to be reasonably close to the OML values. Account has been taken of solar radiation effects and differences arising in the photoemission efficiencies of conductors and dielectric materials. The dynamical consequences of charging on the orbital dynamics of debris have been examined and found to be negligible except for micron and sub-micron sized particles. However the presence of charge may aid detection and tracking of debris

through direct in-situ measurements of the resultant currents or by ground based tracking of charge induced plasma excitations.

The above estimates can be further refined through more sophisticated simulation studies on the SPIS platform to account for irregular shapes of the objects as well as through inclusion of higher order effects influencing ionisation and de-ionisation processes. Another potential area of future research concerns the influence of charge on the drag that a debris object experiences. Our preliminary results, not discussed here, show that the presence of charged micron sized debris can exert a significant drag on a larger sized debris and in effect substantially increase the normal neutral collisional drag that the debris experiences. Such an effect can contribute positively to the efficacy of debris mitigation schemes based on injection of micron sized particles in the orbital region to induce artificial drag on the debris [6]. These and other results pertaining to the testing of the Rogowsky coil concept will be reported at a later date.

VI. ACKNOWLEDGEMENT

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